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Classes of Uniformly Convex and Uniformly Starlike Functions as Dual Sets

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In this paper the classes of uniformly convex and uniformly starlike functions are presented as dual sets for certain function families (in the sense of convolution theory). The results are used to find some sharp sufficient conditions for functions, regular in the unit disk, to belong to the above classes.

1. INTRODUCTION

Denote by A the class of normalized functions $f(z) = z + \sum_{n=2}^{\infty} a_n(f)z^n$, regular in the unit disk $E = \{z : |z| < 1\}$. Consider also its subclasses S, ST, CV, consisting of the univalent, starlike, and convex functions, respectively. It is well known that for any $f \in ST$ not only $f(E)$ but the images of all circles centered at 0 and lying in E are starlike with respect to 0. B. Pinchuk posed a question whether this property is still valid for circles centered at other points of E. A. W. Goodman [1] gave a negative answer to this question and introduced the class UST of uniformly starlike functions $f \in ST$ such that for any circular arc γ lying in E and having the center at $\zeta \in E$ the image $f(\gamma)$ is starlike with respect to $f(\zeta)$. A necessary and sufficient condition for $f \in A$ to be uniformly starlike takes the form

$$\operatorname{Re}\left[(z-\zeta)f'(z)/(f(z)-f(\zeta))\right] > 0, \qquad \forall z, \zeta \in E. \tag{1}$$

In a similar way, the class UCV of uniformly convex functions is defined to include the functions $f \in S$ such that any circular arc γ lying in E with the center $\zeta \in E$ is carried by f into a convex arc. A. W. Goodman [2]

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